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An Integrated Approach to Image Quality: Comparative Analysis of Bilinear and Nearest Neighbor Interpolation

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Abstract

Pixel transitions are critical in image processing, largely depending on interpolation methods to ensure smoothness and clarity. This work focuses on two widely used image interpolation techniques: nearest neighbor interpolation and bilinear interpolation, both implemented using integrated software code. Our methodology enables each interpolation technique to be applied independently, allowing for a direct comparison of their performance. To achieve a thorough evaluation of each interpolation method, we utilize a set of essential quality assessment metrics: Peak Signal-to-Noise Ratio (PSNR), Structural Similarity Index (SSIM), Grayscale Analysis, and Mean Squared Error (MSE). These metrics were selected to provide a balanced assessment of image sharpness, structural accuracy, and overall visual quality. The results of this study offer a detailed analysis of the strengths and limitations of each interpolation technique. These findings are intended to assist researchers and practitioners in selecting the most suitable interpolation method for their specific requirements in the image processing domain. By providing a comparative framework, this work contributes to the field by enhancing methods for assessing and optimizing image quality in digital imaging applications.

Keywords: Image processing, Bilinear interpolation, Nearest neighbor interpolation, Image optimization.

1 | Introduction

Several techniques nearest neighbor interpolation, bilinear interpolation, and cubic interpolation, can scale images. These approaches use various methods to convert input pixel values into output pixel values [1]. The output of each pixel image is given the value depending on the closest pixel in the input image using nearest

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neighbor interpolation. Although quick, this method could produce pixelated or jagged results, especially at high zoom levels.

Bilinear interpolation, on the other hand, determines a pixel's value in the output image by computing the four pixels of a weighted average around it in the input image. It provides smoother results than nearest-neighbor interpolation and is appropriate for various image scaling applications. Each interpolation technique has advantages and is appropriate for certain situations [2], [3]. The most suitable method is chosen based on the particular requirements of the image processing task. However, it is also the most computationally intensive of the three methods [4]. Linear interpolation is a good compromise between speed and quality, while nearest neighbor interpolation is the fastest but results in the lowest quality.

Bilinear interpolation, a commonly used image processing technique, provides smoother pixel transitions during resizing and enhancement. A basic image processing method, nearest neighbor interpolation, offers a straightforward approach to pixel resampling [5]. Typically, a sampled data system representing a reference with an image of a 2D array of the samples is linearly distributed in the x-direction (horizontal). Y direction (vertical). Then, a new group of sampling points or high-resolution pixels is created between reference or input pixels [6]. These are typically high-resolution pixels. Almost everyone has the closest neighbor, four, on a rectangular grid. It is often assumed that a unit square between four adjacent ones. The resolution pixel coordinate for expensive will be x.

To this unit square, add the fraction and y fraction [7]. The question in the interpolation problem is: determine the proper values for these fractions. One of the best ways to solve this problem is to develop the problem Interpolation scheme a detailed description of the interpolation scheme Ratio of High-Res Pixels to Low-Res Pixels. This is effective in key performance indicators such as speed and quality [8]. Image enlargement is one of the basic image manipulations widely used in many applications. Image upscaling is converting a low-resolution image into a high-resolution image. Image enlargement is essentially an image interpolation process. There are many practical image enlargement methods, each with its characteristics, strengths, and weaknesses [9].

The choice of various image enlargement algorithms directly affects the quality of the enlarged image. Therefore, it is important to find suitable algorithms to improve the quality of image enlargement. The most commonly used image enlargement method today is the interpolation method. The following experiment compared the image enlargement performance of diverse interpolation algorithms from subjective and objective points of view so that users can choose the right algorithm to achieve the best results according to their application.

This paper aims to explore the forefront of image enhancement with studies comparing nearest-neighbor and bilinear interpolation and discovering which method is best suited for pixel-perfect visual quality. Discover the mystic behind pristine images by analyzing nearest neighbor and bilinear interpolation techniques and equipping yourself with the knowledge to achieve superior image processing results.

2 | Literature Review

Interpolation remains a crucial tool in digital image processing, where it is used to resize images, smooth pixel transitions, and enhance overall image quality. This process is highly significant in medical imaging, remote sensing, and digital photography, where precise image resolution and accuracy are essential [10]. Studies have shown that the choice of interpolation method directly influences the final visual quality, computational speed, and precision of the processed image. Among the various interpolation methods available, nearest neighbor and bilinear interpolation are the most widely used due to their effectiveness, simplicity, and versatility [11–13].

Nearest neighbor interpolation is one of the simplest techniques, where the value of the closest pixel is used to fill missing values during image scaling. This method is computationally inexpensive, as it involves minimal calculations, making it suitable for applications where speed is prioritized over image smoothness. However,

nearest-neighbor interpolation often produces blocky artifacts and pixelation, particularly when upsampling images [14]. Despite these limitations, it is commonly applied in scenarios with limited processing resources, such as low-resolution displays and certain real-time imaging tasks where image quality is not the primary concern [15].

In contrast, bilinear interpolation is a more advanced method that calculates the average surrounding pixel values to produce a smoother and visually appealing image [16]. Bilinear interpolation considers the weighted average of the four nearest pixel values, resulting in smoother pixel transitions and reducing jagged edges [17]. Although bilinear interpolation is computationally more demanding than nearest neighbor, it produces higher-quality images suitable for applications where both image smoothness and realism are important. This method is frequently used in consumer-oriented products such as digital cameras, image processing software, and display devices, where precision and visual quality are prioritized [18].

Researchers often use objective image quality metrics to conduct a robust comparison of interpolation methods. Peak Signal-to-Noise Ratio (PSNR) is a common metric that evaluates the quality of an image by measuring the ratio of the maximum pixel intensity to the error between the original and reconstructed images [19]. Higher PSNR values indicate lower error levels and correspond to better image quality. The Structural Similarity Index (SSIM), another critical metric, assesses the processed image's quality relative to the original in terms of luminance, contrast, and structural similarity [20]. SSIM is valued for its ability to provide a holistic evaluation of image quality that aligns closely with human visual perception.

In addition to PSNR and SSIM, Mean Squared Error (MSE) and Grayscale Analysis are often employed to evaluate interpolation effectiveness [21] further. MSE calculates the average of the squared differences between pixel values, quantifying the overall error introduced during processing; lower MSE values are desirable as they indicate minimal distortion [22]. Grayscale Analysis, meanwhile, measures how well the interpolation method preserves grayscale intensity values across the image—a critical factor in applications like medical imaging, where subtle intensity variations are important for accurate interpretation [23]. Together, these metrics offer a comprehensive evaluation framework that enables researchers to assess the trade-offs between image quality and computational efficiency across interpolation methods.

Several studies have examined the strengths and limitations of various interpolation techniques, emphasizing the need for methods that balance image quality and processing speed. Nearest neighbor interpolation is favored for its simplicity and rapid execution, although it lacks smoothness in the final image [24]. Bilinear interpolation, while more time-intensive, provides a smoother gradient and higher precision, making it ideal for applications requiring high-resolution images [25]. Recent research has also explored hybrid approaches, which apply different interpolation techniques selectively based on specific image regions or processing tasks to optimize performance [26].

Thus, comparing nearest neighbor and bilinear interpolation techniques offers valuable insights for improving image processing workflows. Researchers can evaluate and compare each method's unique characteristics by employing metrics such as PSNR, SSIM, MSE, and Grayscale Analysis. This review underscores the importance of selecting an interpolation technique suited to the application's specific requirements, whether it demands fast computation, high resolution, or improved structural integrity.

This article presents carefully crafted codes for 2 of the most commonly used image interpolation methods, nearest neighbor interpolation and bilinear interpolation, in an attempt to bridge the gap between theory and practice.

- I. Our contribution goes beyond mere implementation. We provide an integrated code base that allows users to apply these interpolation techniques and perform direct comparative analysis seamlessly. This approach provides a deeper understanding of the technique and helps researchers and practitioners make informed decisions for their specific imaging needs.
- II. We used key image quality criteria, such as PSNR and SSIM, to measure the effectiveness of different approaches.

III. Use MSE: these metrics serve as objective measures and provide insight into the effectiveness of each interpolation method.

Our comprehensive analysis builds on decades of extensive research and provides valuable insight into each interpolation method's strengths and limitations. This is especially important as machine vision applications become more diverse and evolve.

3 | Methodology

Start by acquiring a dataset suitable for your image processing experiment. The dataset should ideally contain different images representing different scenes, objects, and levels of complexity. Download the dataset. Implement the nearest neighbor and bilinear interpolation using a suitable code library (e.g., OpenCV, etc. in Python). Create a separate code script for each interpolation method to ensure clarity and modularity. Compute the SSIM and MSE for each resized image using nearest neighbor and bilinear interpolation [27]. PSNR measures a commonly used indicator of picture fidelity that compares the quality of a reduced image to the original. Gather the PSNR, MSE, and SSIM values acquired for each image in the collection [28]. We compare the performance of nearest neighbor and bilinear interpolation techniques based on the computed MSE, PSNR, and SSIM values and use a flow chart to illustrate the methodology.

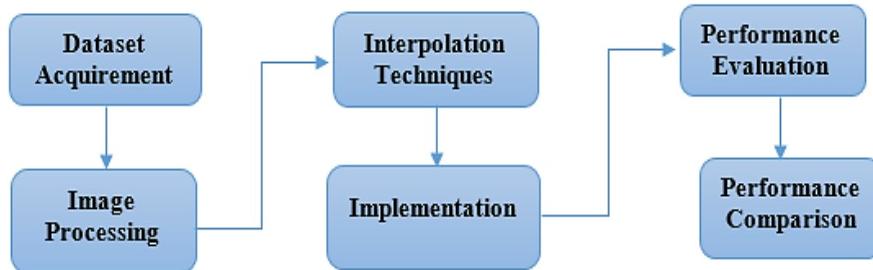


Fig. 1. Illustrates the methodology.

3.1 | Experiment Environment

For the experiment setup, the following hardware and software were used. Use a computer with an image processing library (such as OpenCV, Numpy, or Pandas). Choose Python as the programming language, and use the Notebook for algorithm implementation and data analysis. Use MathCast to write the equation for the calculation. For Datasets and Interpolation, prepare diverse image data sets, implement bilinear and nearest-neighbor interpolation algorithms, and run experiments to assess the PSNR, MSE, and SSIM.

3.2 | Metrics for Performance Evaluation

The Metrics for performance evaluation for both interpolations are SSIM, MSE, and PSNR. There are many reasons for using MSE the reason for using MSE quantifies the mean squared difference between the original and interpolated pixel values. The structural similarity between the interpolated and original images is measured using SSIM. By contrasting the interpolated image with the actual image, PSNR calculates quality.

3.3 | Equation for Structural Similarity Index

Calculate the SSIM between the original image and each interpolated image. Check your window size (win_size) used in SSIM calculations is suitable for image size.

The SSIM formula for two images, I (original) and K (interpolated) is:

$$\text{SSIM}(I, K) = \frac{(2\mu_I\mu_K + C_1)(2\sigma_{IK} + C_2)}{(\mu_I^2 + \mu_K^2 + C_1)(\sigma_I^2 + \sigma_K^2 + C_2)}. \quad (1)$$

- I. μ_I^2 and μ_K^2 are the average pixel values of images I and K.
- II. σ_I^2 and σ_K^2 are the variances of images I and K.
- III. σ_{IK} is the covariance between I and K.
- IV. C_1 and C_2 are constants to stabilize the division when the denominator is small (often set as $C_1 = (0.01 \cdot \text{MAX})^2$ and $C_2 = (0.03 \cdot \text{MAX})^2$).

Choosing window size (win_size) for SSIM

The SSIM calculation can be performed using a sliding window approach, where a small window (e.g., 11×11) is moved across the image. The window size should be chosen based on the resolution and details of the image; larger windows may better capture structural similarity in high-resolution images, while smaller windows are useful for capturing fine details in lower-resolution images.

3.4 | Equation for Mean Squared Error

The equation used to calculate the MSE. Calculate the MSE between the original and interpolated images using the following formula.

$$\frac{1}{nm} \sum_{i=1}^n \sum_{j=1}^m [I(i,j) - K(i,j)]^2. \quad (2)$$

- I. The original image is I.
- II. K is the interpolated image.
- III. The number of rows is n.
- IV. The number of columns is m.
- V. Represented pixel values I(i,j) and K(i,j).

3.5 | Equation for Peak Signal-to-Noise Ratio

The PSNR between interpolated and original images is determined using the formula.

$$\text{PSNR}(I, K) = 10 \cdot \log_{10} \left(\frac{\text{MAX}^2}{\text{MSE}(I, K)} \right). \quad (3)$$

Where the maximum pixel value is MAX that can be used. Value (for instance, 255 for 8-bit images).

Table 1. Information of Dataset.

Interpolation Type	Dataset Images Quality	Number of Images	Formats
Nearest neighbor	Used mix blur, Clear	300-500	JPG, PNG
Bilinear	Used mix blur, Clear	300-500	JPG, PNG

The Dataset statistics are shown in *Table 1*. The information provided about the data has been used under different conditions.

4 | Results

Results suggest that Bilinear interpolation outperforms nearest-neighbor interpolation from an SSIM perspective, indicating better structural similarity between original and interpolated images. In addition, bilinear interpolation results in lower MSE and smaller pixel-wise differences between images. Therefore,

based on the analysis, we believe that bilinear interpolation is the better choice to achieve smoother pixel transitions during image processing.

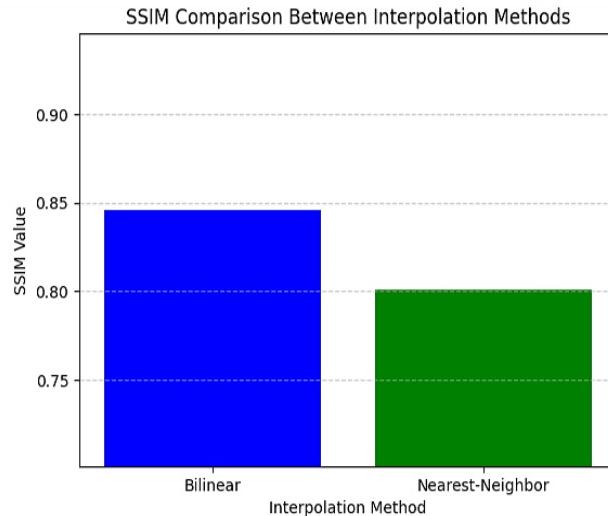


Fig. 2. Result of SSIM.

In Fig. 2, the Bar graph compares the SSIM values between the two interpolation methods. Bilinear interpolation Higher achieves smoother pixel transitions during image processing.

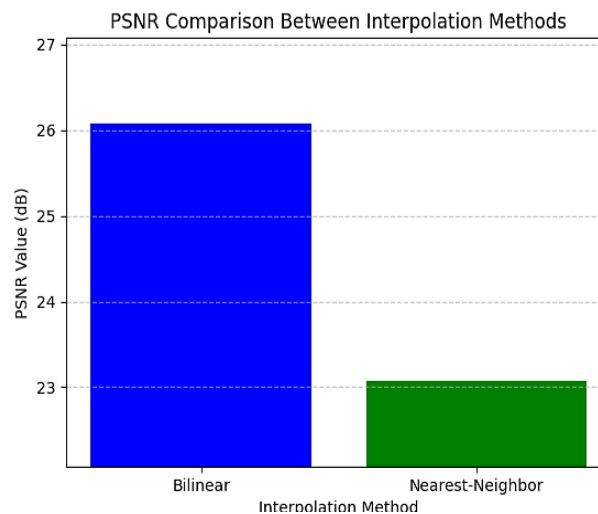


Fig. 3. Result of PSNR.

In Fig. 3, the Bar graph compares the PSNR values between the two interpolation methods. A higher Bilinear interpolation PSNR value shows that bilinear interpolation performs better regarding image quality for preservation.

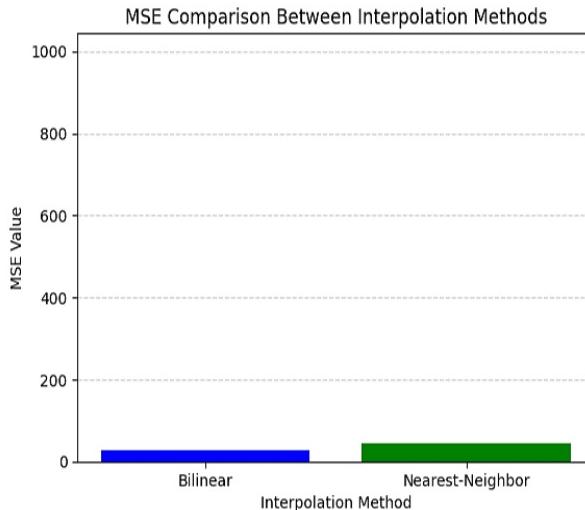
**Fig. 4. Result of MSE.**

Fig. 4, the Bar graph compares the MSE values between the two interpolation methods. Lower MSE values of bilinear interpolation indicate that bilinear interpolation has better image quality, so bilinear interpolation performs better.

Table 2. Result of Interpolation.

Interpolation Type	Matrix Result		
	Mean Squared Error	Peak Signal-to-Noise Ratio	Structural Similarity Index
Nearest_Neighbor	28.60	26.07	0.8456
Bilinear	44.22	23.07	0.8011

Table 2 showed that for bilinear interpolation, the SSIM value was 0.8456, PSNR 23.07, and the MSE was 28.6043. In contrast, nearest neighbor interpolation yielded an SSIM value of 0.8011, PSNR of 26.07, and MSE of 44.2287.

According to the greyscale analysis result here we provided different dimensions.

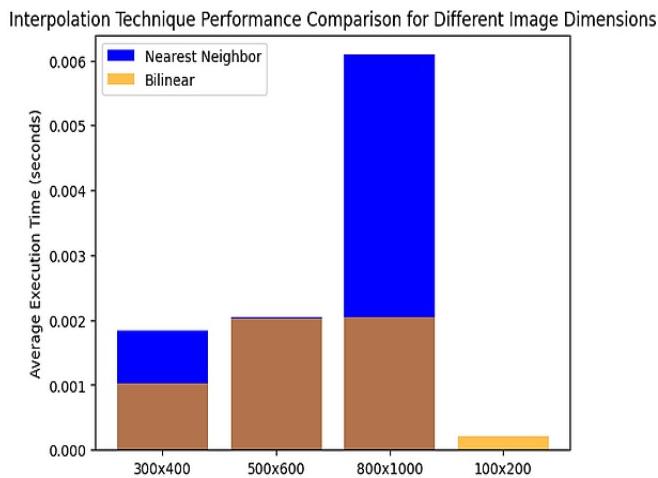
**Fig. 5. Result of greyscale analysis.**

Fig. 5 Bar graph compares the Greyscale analysis values between the two interpolation methods. A higher Bilinear interpolation analysis value shows that bilinear interpolation performs better regarding image quality for preservation.

Table 3. Result of Greyscale analysis.

Target Dimensions	Nearest Neighbor Interpolation (Avg. Execution Time)	Bilinear Interpolation (Avg. Execution Time)
(300, 400)	0.0018 seconds	0.0010 seconds
(500, 600)	0.0020 seconds	0.0020 seconds
(800, 1000)	0.0061 seconds	0.0020 seconds
(100, 200)	0.0 seconds	0.0002 seconds

Table 3 shows the average execution durations for bilinear and nearest-neighbor interpolation for various target dimensions are compared in this table. We have rounded the figures for ease of use.

The variations in execution times are negligible for images with lower dimensions, such as (300, 400) and (100, 200). However, nearest neighbor interpolation might be slightly faster for really small images. While bilinear interpolation is generally steady, nearest neighbor interpolation experiences a considerable increase in execution time as image size increases. In this investigation, bilinear interpolation performs better overall and is more consistent regarding execution time across various image sizes. It's crucial to remember that the precise performance characteristics can change depending on the software and hardware environment used to run the code. The conclusions you've supplied are based on your provided data; therefore, real outcomes may differ in many aspects.

Bilinear interpolation and nearest-neighbor interpolation, two frequently used picture interpolation techniques, were thoroughly examined in this study. Four important image quality metrics—SSIM, PSNR, and MSE—were used to compare the two methods to identify which one delivers superior results regarding image quality. Two well-known image interpolation methods evaluated Bilinear interpolation and nearest neighbor interpolation. These findings imply that, from an SSIM standpoint, bilinear interpolation beats nearest-neighbor interpolation, suggesting improved structural similarity between original and interpolated pictures. Additionally, bilinear interpolation produces reduced pixel-wise variations between images and lower MSE. As a result, we think that bilinear interpolation is a preferable option to generate more seamless pixel transitions during image processing based on the analysis we performed. Researchers and industry experts will gain important knowledge from this thorough study, which will aid them in selecting the best.

4.1 | Future Research Directions

The study of interpolation and its application in decision-making processes through fuzzy set extensions offers substantial opportunities for further investigation. Advanced fuzzy set theories and their expansions, including Pythagorean fuzzy sets, Q-rung orthopair fuzzy sets, neutrosophic fuzzy sets, and hypersoft sets, can improve decision-making models in intricate contexts. Future research directions are offered based on insights from studies [29-50].

- I. Pythagorean fuzzy sets offer increased adaptability in addressing uncertainty and ambiguity, as discussed in [29], [34], [35], [40]. Interpolation techniques designed for these Datasets can enhance the resilience of decision models in multi-criteria contexts.
- II. Applying Q-rung orthopair fuzzy sets, as examined in [32], [37–39], provides enhanced flexibility in depicting intricate decision-making situations. Creating interpolation models for these Datasets may yield more precise results in medical diagnostics [37] and energy systems [47].
- III. As illustrated in [30], [31], [33], and [35], the neutrosophic hypersoft set framework can use interpolation techniques to address group decision-making issues characterized by varied and competing criteria.
- IV. The advancement of picture interpolation methods utilizing Pythagorean or Q-rung orthopair fuzzy sets may resolve issues in image enhancement and noise reduction [36], [45], [50].

- V. Fuzzy logic can be utilized to develop resilient cryptographic systems, as evidenced in [36], [42], [43], [46], [48], and [49]. Future studies may investigate fuzzy interpolation for dynamic key generation in encryption methods to improve security against cyberattacks.
- VI. Expanding the application of fuzzy logic in AI-enhanced educational systems, as demonstrated in [41] and [48], has the potential to transform information processing and decision-making in academic environments.
- VII. As proposed in [44] and [50], integrating fuzzy set theories into deep learning models may enhance interpretability and robustness in applications such as text readability assessment and trading risk evaluation.
- VIII. Fuzzy systems can be applied to political education to tackle issues in policy-making and governance, as outlined in [41].
- IX. Employing fuzzy hypersoft sets for energy storage prioritization [38] and supplier selection [47] may facilitate the transition to sustainable energy systems.

Future research can substantially enhance the applicability of fuzzy set extensions and their role in decision-making, as outlined in references [29-50]. Researchers can create robust and scalable solutions for complicated decision-making problems by using fuzzy set extensions to advanced interpolation techniques, incorporating AI, and tackling real-world challenges across several fields. These directions provide a means to connect theoretical developments with actual applications, promoting innovation in fuzzy decision-making.

5 | Conclusion

This study comprehensively analyzed two widely used image interpolation techniques: bilinear interpolation and nearest neighbor interpolation. The primary objective was to evaluate the effectiveness of each method in producing high-quality images, using PSNR, SSIM, and MSE as key metrics. These metrics provided a quantitative basis for comparing the benefits and limitations of each interpolation approach. Our analysis yielded several significant observations. Bilinear interpolation demonstrated clear advantages, particularly in structural preservation, as evidenced by its higher SSIM values. This indicates that bilinear interpolation effectively maintains the original image's structural integrity, resulting in smoother transitions and reducing the "staircase" effect commonly seen in lower-quality interpolations.

Additionally, bilinear interpolation consistently produced lower MSE values, reflecting its superior capability to minimize pixel-level discrepancies and create visually coherent, smooth images. These findings position bilinear interpolation as the preferred technique in applications with critical pixel transitions and structural continuity. Beyond the technical evaluation, this research has practical implications for researchers, professionals, and industry stakeholders in fields that demand accurate image analysis. By clarifying the strengths and weaknesses of each interpolation method, this study provides valuable insights to guide the selection of an appropriate approach based on specific application requirements. For example, nearest-neighbor interpolation may be ideal for real-time applications where processing speed is paramount.

In contrast, bilinear interpolation is recommended for applications requiring high visual fidelity, such as digital cameras, medical imaging, and display systems. Thus, this research bridges the gap between theoretical insights and practical applications in image processing, affirming that bilinear interpolation offers superior image quality and structural accuracy. This technique is, therefore, well-suited for a broad range of image processing applications, and its effective use will enable practitioners to optimize image quality as advancements in digital imaging continue. As technology in image processing continues to advance, the insights from this study will enable practitioners to optimize their workflows, enhance image clarity, and deliver improved results across various imaging applications.

Author Contributions

Seher Ilyas contributed to the conceptualization and design of the study and the development of the comparative framework. Wusat Ullah was involved in the data analysis, image processing, and the implementation of bilinear and nearest neighbor interpolation methods. Hamza Naveed contributed to the

literature review and experimental setup, while Saalam Ali supported data validation and manuscript writing. All authors participated in the critical revision of the manuscript and approved the final version.

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Data Availability

The datasets and code generated during this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare no conflicts of interest regarding this study.

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